AD-A250 175

[REPORT DOCUMENTAT]ION PAGE

Form Approved
OMB No. 0704-0188

(2)

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE

3. REPORT TYPE AND DATES COVERED
FINAL 15 OCT 90-29 NOV 91

4. TITLE AND SUBTITLE
"SYMMETRY METHODS & NONLINEAR ANALYSIS IN ELASTOMECHANICS" (U)

5. FUNDING NUMBERS
61102F
2304/A9

6. AUTHOR(S)
Dr. Timothy J. Healey

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Cornell University
Center for Applied Mathematics
Ithaca, NY 14853

8. PERFORMING ORGANIZATION REPORT NUMBER
AFOSR-TR 92 0370

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AFOSR/ NM
Bldg 410
Bolling AFB DC 20332-6448

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
AFOSR-91-0062

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for Public Release;
Distribution Unlimited

12b. DISTRIBUTION CODE
UL

13. ABSTRACT *(Maximum 200 words)*

The essential goal of this investigation was to combine symmetry-reduction techniques with methods of nonlinear analysis to solve nonlinear differential equations associated with problems from elastomechanics. During the effective time of this grant papers have appeared and papers have been accepted for publication.

14. SUBJECT TERMS

15. NUMBER OF PAGES
6

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR

FINAL REPORT ON GRANT AFOSR–91–0062

Symmetry Methods and Nonlinear Analysis in Elastomechanics

Timothy J. Healey

Department of Theoretical & Applied Mechanics and

Center for Applied Mathematics

Cornell University

Ithaca, NY 14853

March 18, 1992

CONTENTS

1. Introduction

The essential goal of this investigation was to combine symmetry–reduction techniques with methods of nonlinear analysis to solve nonlinear differential equations associated with problems from elastomechanics. During the effective time of this grant, October 15, 1990 – December 30, 1991, papers [H1]–[H7] (cf. Sec. 4) have appeared, and papers [H8]–[H10] have been accepted for publication. A description of the major results obtained, which include significant breakthroughs associated with nodal properties of elliptic PDE in global bifurcation problems, is given below.

2. Description of Results

In paper [H1] the existence of large rotating states of a current–carrying, elastic wire in the presence of an ambient magnetic field is established. The main point of the paper is that the governing equations are equivariant under a subtle representation of $O(2) \subset SO(3)$. The subtlety arises from the fact that the Lorenz force breaks all reflection symmetries; yet, a "flip" symmetry is retained. This identification enables a local and global analysis of the helically–symmetric nontrivial states. This paper serves as a paradigm for more general problems of magneto–elasticity.

The stability and bifurcation of a freely rotating circular loop of elastic string is studied in [H2]. Employing the energy–momentum method of Routh, we first show that the circular relative–equilibrium shape is unconditionally stable (for all values of prescribed angular momentum) for inextensible strings and stiff elastics strings. However, the stability test fails for soft elastic strings above a certain critical value of the angular momentum. A bifurcation analysis then shows that a rich assortment of noncircular solutions are possible (within the confines of physically reasonable constitutive hypotheses). In particular, we show that an isola bifurcation (universal Z_2–unfolding) occurs for a broad class of materials often associated with rubber. In paper [H5], we show that this same type of bifurcation occurs in inflated, spherical, rubber balloons. This result agrees with the experimentally observed fact that such membranes typically possess a stable phase of nonspherical, axisymmetric equilibrium for a certain small range of the control parameter (e.g., the total mass of the enclosed gas).

Paper [H3] demonstrates the efficient computational use of the well known fact that a square matrix can always be block–diagonalized (via a similarity transformation) whenever it commutes with a group of square matrices (of the same dimension). In particular, this is the case for the mass and (linearized) stiffness matrices of structural systems with symmetry, which commute with a representation of the symmetry group. The main point of the paper is two–fold: (i) to demonstrate the utility of such techniques to the structural/computational mechanics community; (ii) to show that the "blocks" can be assembled directly (via the matrices of a repeating substructure) without explicitly performing the similarity transformation. This paper has immediate applications to nonlinear bifurcation problems.

Paper [H4] contains the first results on nodal properties for elliptic PDE in global bifurcation problems. There we consider two–dimensional domains with rectangular (orthotropic) symmetry. By exploiting the symmetry, we work in reduced Banach spaces in which families of nodal lines, which coincide precisely with those of the eigenfunction of the linearized problem, are frozen along entire global solution branches. By appealing to subtle maximum principles, we then show that no new nodal sets can be "born" away from the trivial solution. In a major breakthrough, we generalize these results to more exotic symmetries and to n–dimensional domains ($n \geq 2$) in [H10]. Indeed, the restriction to rectangular domains in $\mathbb{R}^2$ in [H4] follows from the use of a well–known boundary–corner lemma to the maximum priciple. In [H10] we are confronted with acute–angle corners in $\mathbb{R}^2$ and higher–dimensional corners, at which the order of the zero is indeterminant, and hence no such boundary lemma exists. Instead we show indirectly via an entirely new approach, that no new nodal sets can be "born" out of such corners. This follows from the contra– diction of a standard Poincare inequality with a Cacciapoli inequality over any new (necessarily small) nodal domain. In paper [H8] we establish the existence of an unbounded, global, bifurcating branch of positive solutions of fully nonlinear elliptic PDE on arbitrary domains having sufficient boundary regularity. In [H7] and [H11] we demonstrate that solutions obtained in [H4] and [H10], respectively, also satisfy fully nonlinear, homogeneous, Neumann boundary conditions on certain domains that are complementary to nodal domains.

By group–theoretic reasoning alone, we deduce in [H9] a new class of free, large–amplitude oscillations of a fixed, cantilevered, Cosserat rod. During such motions the deformed centerline of the rod rotates steadily about the vertical, while each cross section "wobbles" with a distinct angular velocity (the base is fixed, and hence the motion is not rigid). We present a new formulation for such problems which enables a straightforward identification of the spatio–temporal symmetry group. The symmetry reduction yields a two–point boundary value problem, the global analysis of which we present in the remainder of the work.

In [H6] we present local existence results for a general class of live, axisymmetric, traction problems from 3–dimensional, nonlinear elasticity. A group–theoretic reduction removes the usual difficulties associated with the nonlinear compatibility conditions for the boundary tractions, and we obtain existence via the implicit–function theorem.

3. **Conclusions**

The most significant results of this project are those dealing with the connection between symmetry and nodal structure of global solutions of differential equations, cf. [H4], [H10]. In this way we were able to generalize the classical work of Crandall and Rabinowitz [1] (on nonlinear Sturm–Liouville eigenvalue problems) to elliptic PDE.

[1] M.G. Crandall & P. Rabinowitz, J. Math. Mech. **19** (1970) 1083–1102.

4. Publications

[H1] Large Rotating States of a Conducting Elastic Wire in a Magnetic Field: Subtle Symmetry and Multiparameter Bifurcation, J. Elasticity **24** (1990) 211–228.

[H2] Stability and Bifurcation of Rotating Nonlinearly Elastic Loops, Quart. Appl. Math. **48** (1990) 679–698.

[H3] Exact Block Diagonalization of Large Eigenvalue Problems for Structures with Symmetry (with J. Treacy), Inter. J. Num. Meth. Engrg. **31** (1991) 265–285.

[H4] Symmetry and Nodal Properties in Global Bifurcation Analysis of Quasi–Linear Elliptic Equations (with H. Kielhofer), Arch. Rat. Mech. Anal. **113** (1991) 299–311.

[H5] Bifurcation to Pear–Shaped Equilibria of Pressurized Spherical Membranes (with Y.–C. Chen), Int. J. Nonl. Mech. **26** (1991) 279–291.

[H6] Local Solutions of the Traction Problem for Elastic Aeolotropic Bodies of Revolution: Symmetry Reduction and Continuation, in Proc. ASME Winter Annual Meeting, Atlanta, Dec. 1991.

[H7] Hidden Symmetry of Fully Nonlinear Boundary Conditions in Elliptic Equations: Global Bifurcation and Nodal Structure (with H. Kielhofer), Results in Math. 21 (1992) 833–92.

[H8] Positivity of Global Branches of Fully Nonlinear Elliptic Boundary Value Problems (with H. Kielhofer), Proc. AMS (1992, in press).

[H9] Large Rotatory Oscillations of Nonlinearly Elastic Rods: Spatio–Temporal Symmetry–Breaking Bifurcation, SIAM J. Appl. Math (1992, in press).

[H10] Preservation of Nodal Structure on Global Bifurcating Solution Branches of Elliptic Equations with Symmetry (with H. Kielhofer), J. Diff. Eq. (1992, in press).

[H11] Symmetry and Preservation of Nodal Structure in Elliptic Equations Satisfying Fully Nonlinear Neumann Boundary Conditions (with H. Kielhofer), to appear Proc. Bifurcation and Symmetry: Cross Influences Between Mathematics and Applications, Marburg, Germany, 1991.